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# Determination of Material for Shaft Design Using on Grey Correlation Analysis and TOPSIS Method

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**ABSTRACT**
Machines, automobiles, aircrafts and many other applications have shaft as major mechanical component which must have a proper design, in-order to have the efficient transmission of power from one element to another. For the design of shaft an appropriate range of evaluation, general product form and processing methods for material must be made. The selection of material should be done by using multiple attribute decision methods (MADM). In this paper, Grey Correlation Analysis and TOPSIS Method is proposed in order to decide a suitable material by considering different attributes and graphical representations are made for different attributes verse materials and vice versa.
***Keywords* -** Shaft; Materials; Design constraints; Grey Correlation Analysis, TOPSIS Method

## I. INTRODUCTION

The Shaft transfers the power from one end to another, thus they subjected to torque and are to be acclaimed from axles for support rotating moments Material Engineering mainly focuses on the optimization of materials according to different applications in high-end. Considering different properties and characteristics of material modelling is essential in relating mechanical, chemical and thermal impacts [1-2]. According to the properties only the way of material will change in the usage for a specific application, materials like metals, alloys, polymers, ceramics and high-ended properties based materials will give reliability and lifespan of components during operation. In recent years Shaft design enterprises have experienced unprecedented crisis of quality credit. The quality credit is gradually put onto the cusp as a crucial social issue. Some companies collapse instantly owing to their terrible quality credit while others struggle to survive in this battle. In order to strengthen the internal and external regulatory efficiency of the enterprises, comprehensively improve the quality of products, reduce the safety risk in some areas, and improve people's satisfaction at the same time, to improve credit system construction and establish a scientific and reasonable credit evaluation system is the current urgent need in our country.

A lot of domestic scholars have already done some researches on quality credit evaluation. The classical MCDM method technique for order preference by similarity to ideal solution (TOPSIS) was used by ZHU [3]. However, AHP, which is consisted of more qualitative ingredient, needs too many large amounts of data statistics and weights to easily determine. TOPSIS can't reflect the difference between internal factors' change trend and the ideal solution well. Some scholars have tried to combine TOPSIS with grey correlation analysis so that the combination can more accurately describe the integration degree of alternative and ideal schemes, and used to compare the superiority of alternative schemes [4-6]. Now this paper attempts to use the combination of Grey Correlation Analysis and TOPSIS Method For the analysis of multiple attributes to solve material used for design of shaft with multi attribute optimization

## II. SHAFT MODEL

Shafts are usually cylindrical machine component that transmits power. The shaft is designed on the basis of strength and its rigidity and stiffness. Shafts are subjected from the axles but only to bending loads and will not transfer power and torque [7-9]. The Figure 1 represents the materials used in manufacturing industries from BC to date. The Shaft is linked with different ways which will be flexible. As torque transmitted by shaft remains constant for a long time, the shearing stress on the shaft cross-section changes much less frequently. Every System will have regulations for making specialized and standardization a product. For the better gains and good product along with above attributes Laboratory analysis must be made for the product.

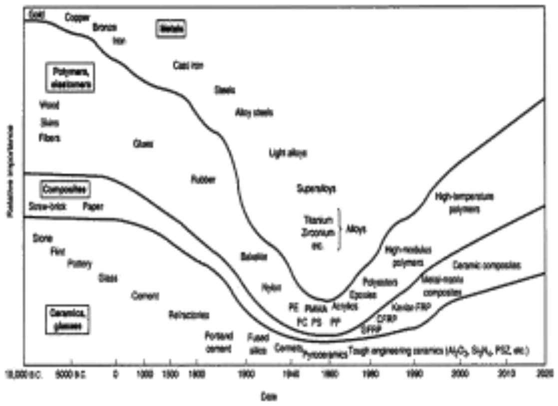

**Fig.1 Materials from BC to date**

For the better gains and good product mechanical properties play a major role [10]. In metals, the properties which has to satisfy as per the standard values are Modulus of elasticity, Ductility, Fatigue strength, Impact strength, Coefficient of thermal expansion, Density, Yield strength, Shear strength, Tensile strength, Thermal conductivity, toughness etc and in polymers, the properties like Stability, Stiffness, Chemical, absorption & electrical resistance etc.

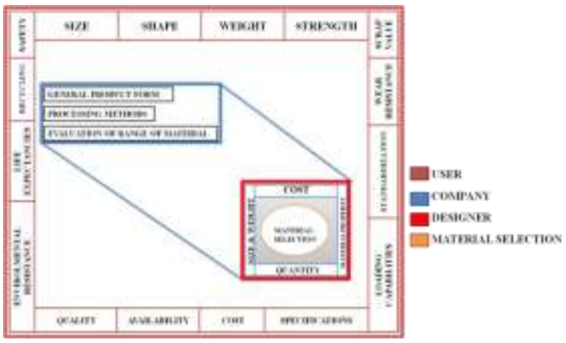


**Fig.2 Analytical Process for Selection of Material for Design**

Above Fig.2 is an analytical approach for selection of material. Initial Considerations are done according to the Users and Consumers, in the above fig all the attributes are represented as per user's requirements [11]. According to the User desire and company's availability among all three attributes are considered at company levels. Company will assign the needs to design engineer, for the better product output design will consider four attributes as shown in the figure. Considering all the attributes selection of material is done for any product manufacturing (shaft)

## III. PROPOSED METHODOLOGY

For the Evalution of best material for the design of shaft this paper uses the improved TOPSIS model by grey correlation analysis. The Proposed Method is as follows:

**Step 1:** In the first step, we have to determine the objective and to identify the attribute values for each material.

**Step 2:** This step involves the development of matrix formats. The decision making matrix can be expressed as:

$$D = \begin{bmatrix} x_{11} & x_{12} & x_{13} & \cdots & x_{1n} \\ x_{21} & x_{22} & x_{23} & \cdots & x_{2n} \\ x_{31} & x_{32} & x_{33} & \cdots & x_{3n} \\ : & : & : & : & : \\ . & . & . & . & . \\ x_{m1} & x_{m2} & x_{m3} & \cdots & x_{mn} \end{bmatrix}$$

**Step 3:** Then using the above matrix to develop the normalized decision matrix with the help of the formula given below:

$$\boldsymbol{X_{ij}^{*} = X_{ij} \Big/ \sum_{i=1}^{n} X_{ij}}$$

**Step 4:** Depending upon the relative importance of different attributes obtain weight for each attributes using the formula given below and the sum of the weights should be 1. Where $V_j$ is the variance of each attribute which can be calculated by the formula given as

$$\boldsymbol{W_j = V_j \Big/ \sum_{j=1}^{m} V_j} \; ,$$

$$\boldsymbol{V_j = (1/n) \sum_{i=1}^{n} \left(X_{ij}^{*} - (X_{ij}^{*})_{mean}\right)^2}$$

**Step 5:** Then obtain the weighted, normalized matrix $v_{ij}$ by multiplying $W_j$ with all the values $X_{ij}^{*}$ such as equcations

a) The Ideal solution

$$\boldsymbol{A^{+} = \{v_1^{+}, \dots \dots \dots, v_m^{+}\}}$$
$$\boldsymbol{= \{(max v_{ij} \backslash j \epsilon I'), (min v_{ij} \backslash j \epsilon I'')\}}$$

b) The negative ideal solution

$$\boldsymbol{A^{-} = \{v_1^{-}, \dots \dots \dots, v_m^{-}\}}$$
$$\boldsymbol{= \{(min v_{ij} \backslash j \epsilon I'), (max v_{ij} \backslash j \epsilon I'')\}}$$

Here,
I'= {j=1, 2 … n | j}: Associated with the beneficial attributes
I"= {j=1, 2… n | j}: Associated with non-beneficial adverse attributes

**Step 6:** Obtain separation (distance) of each alternative from the ideal solution and negative ideal solution which is given by the Euclidean distance given by the equations:

$$\boldsymbol{D_i^{+} = \sqrt{\sum_{j=1}^{m} \left(v_{ij} - v_j^{+}\right)^2}, i = 1, 2, \dots n}$$

$$\boldsymbol{D_i^-} = \sqrt{\sum_{j=1}^{m}\left(\boldsymbol{v_{ij}} - \boldsymbol{v_j^-}\right)^2}, \boldsymbol{i = 1, 2, \ldots n}$$

**Step 7:** Calculate grey correction degree based on the normalized matrix Y, the grey correction degree of the $i_{th}$ enterprise to the $j_{th}$ indicator of PIS and NIS are [12-14]:

$$\boldsymbol{s_{ij}^+} = \frac{\min\limits_i \min\limits_j \left|v_{ij} - v_j^+\right| + \rho \max\limits_i \max\limits_j \left|v_{ij} - v_j^+\right|}{\left|v_{ij} - v_j^+\right| + \rho \max\limits_i \max\limits_j \left|v_{ij} - v_j^+\right|}$$

$$\boldsymbol{s_{ij}^-} = \frac{\min\limits_i \min\limits_j \left|v_{ij} - v_j^-\right| + \rho \max\limits_i \max\limits_j \left|v_{ij} - v_j^-\right|}{\left|v_{ij} - v_j^-\right| + \rho \max\limits_i \max\limits_j \left|v_{ij} - v_j^-\right|}$$

Normal range of $\boldsymbol{\rho}$ is in the interval [0, 1]. According to experience, let $\boldsymbol{\rho}$ =0.5 in this access. Relational matrix can be expressed as:

$$W^+ = \begin{bmatrix} s_{11}^+ & s_{12}^+ & s_{13}^+ & \cdots & s_{1n}^+ \\ s_{21}^+ & s_{22}^+ & s_{23}^+ & \cdots & s_{2n}^+ \\ s_{31}^+ & s_{32}^+ & s_{33}^+ & \cdots & s_{3n}^+ \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ s_{m1}^+ & s_{m2}^+ & s_{m3}^+ & \cdots & s_{mn}^+ \end{bmatrix}$$

$$W^- = \begin{bmatrix} s_{11}^- & s_{12}^- & s_{13}^- & \cdots & s_{1n}^- \\ s_{21}^- & s_{22}^- & s_{23}^- & \cdots & s_{2n}^- \\ s_{31}^- & s_{32}^- & s_{33}^- & \cdots & s_{3n}^- \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ s_{m1}^- & s_{m2}^- & s_{m3}^- & \cdots & s_{mn}^- \end{bmatrix}$$

The grey correction degree of the $i_{th}$ enterprise to PIS and NIS are:

$$W_i^+ = \frac{1}{n}\sum_{j=1}^{n} s_{ij}^+, i = 1,2, \ldots, m$$

$$W_i^- = \frac{1}{n}\sum_{j=1}^{n} s_{ij}^-, i = 1,2, \ldots, m$$

**Step 8:** calculate the relative closeness, realize the best rank of $\boldsymbol{m}$ enterprises. Dimensionless Euclidean distance and grey correction degree are respectively represented as $D_i^+, D_i^-, W_i^+, and\ W_i^-$

$$T_i^+ = e_1 D_i^- + e_2 W_i^+, i = 1,2, \ldots m$$

$$T_i^- = e_1 D_i^+ + e_2 W_i^-, i = 1,2, \ldots m$$

Where $\mathbf{e_1}$ and $\mathbf{e_2}$ reflect the degree of decision-makers' preference, and $\mathbf{e_1}$ + $\mathbf{e_2}$=1 Where $T_i^+$and $T_i^-$respectively represents the approaching degree from the $i_{th}$ enterprise to PIS and NIS.

$$\xi_i = {T_i^+}/{(T_i^+ + T_i^-)} \quad i = 1,2, \ldots n$$

Where $\xi_i$is called best quality score in this study representing the relative approaching degree of the $i_{th}$ enterprise to PIS and NIS

**Step 9:** Rank the materials according to their quality scores.

According to the result we get from step 8, rank the materials by their quality scores. When $\xi_i$is bigger, its enterprise is closer to the positive ideal, and vice versa.

## IV. DISCUSSIONS

For the selection of material data is collected for a few materials as shown in Table 1. Step by step procedure is done for the proposed methodology as shown in Appendix. The graphical representations Figure 3& 4 will give a clear idea for the best material among individual attributes for the collected data. According to the collected data of different materials with specific attributes, the proposed methodology is applied and decision for the selection of material in designing a shaft.

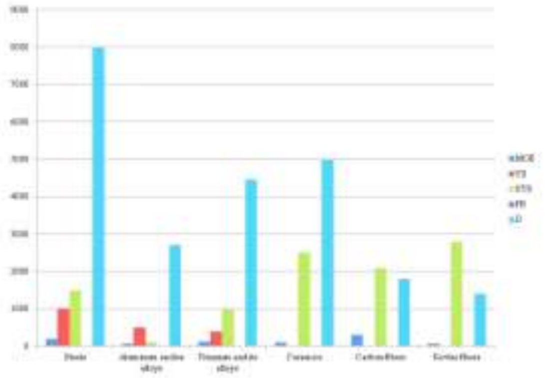

**Fig.3 Graphical representation for Collected Data**

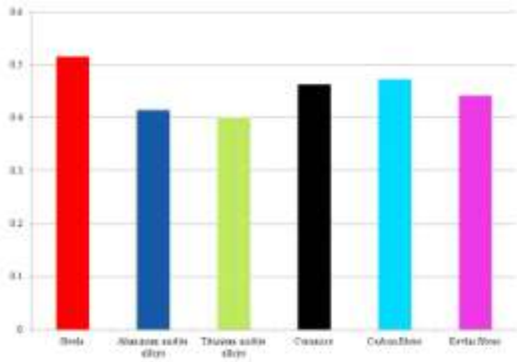

**Fig.4 Graphical representation for Result of Proposed Method**

## V. CONCLUSION

In this paper, Grey Correlation Analysis and TOPSIS Method is applied for the selection of material for the design of the shaft. As the shaft is the major device in the smooth running of a machine. Since the proposed methodology, data are collected as per the sources given by manufacturing industries for each material. Among all the materials, steels, aluminum alloys, Titanium alloys, Ceramics, Carbon Fibers and Kevlar Fibers are considered with attributes Modulus of Elasticity (E) (GPA), Yield Stress (Y) (MPa), Ultimate Tensile Strength (UTS) (MPa), Poisson's ratio (ν) and Density (ρ). Analysis is done for having a decisive material as result for the design of the shaft for the smooth controlling of any machine according to the application. On the other hand, with the change of materials according to the applications we can get a decisive material for the application as the proposed methodology give the best decision in selecting of material. Graphical representations and discussions also demonstrated that materials which are used for the design of the shaft can be selected or decided for manufacturing of the shaft.

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## Appendix

**Table 1: Collected data for selection of material**

| Materials | MOE | YS | UTS | PR | D |
|---|---|---|---|---|---|
| **Steels** | 195 | 1000 | 1500 | 0.32 | 8000 |
| **Aluminum and its alloys** | 65 | 500 | 100 | 0.33 | 2700 |
| **Titanium and its alloys** | 120 | 400 | 1000 | 0.34 | 4450 |
| **Ceramics** | 100 | 0 | 2500 | 0.2 | 5000 |
| **Carbon fibers** | 300 | 0 | 2100 | 0.22 | 1800 |
| **Kevlar fibers** | 70 | 0 | 2800 | 0.36 | 1400 |

MOE=Modulus of Elasticity (E) (GPa), YS=Yield stress (Y) (MPa), UTS=Ultimate Tensile Strength (UTS) (MPa), PR=Poisson's ratio (ν), D=Density (ρ) (kg/m3)
For the Table 1 normalization is done and normalized decision matrix is computed

**Table 2: Normalized Decision matrix**

| Materials | MOE | YS | UTS | PR | D |
|---|---|---|---|---|---|
| **Steels** | 0.4851556 | 0.842151921 | 0.3215598 | 0.4335024 | 0.7263 |
| **Aluminum and its alloys** | 0.1617185 | 0.421075961 | 0.02143733 | 0.44704942 | 0.24518 |
| **Titanium and its alloys** | 0.2985573 | 0.336860768 | 0.21437321 | 0.4605963 | 0.40407 |
| **Ceramics** | 0.2487977 | 0 | 0.53593309 | 0.2709390 | 0.45397 |
| **Carbon fibers** | 0.7463933 | 0 | 0.45018376 | 0.2980329 | 0.16349 |
| **Kevlar fibers** | 0.1741584 | 0 | 0.60024508 | 0.4876902 | 0.12719 |

MOE=Modulus of Elasticity (E) (GPa), YS=Yield stress (Y) (MPa), UTS=Ultimate Tensile Strength (UTS) (MPa), PR=Poisson's ratio (ν), D=Density (ρ) (kg/m3)
From Table 2 weighted normalized decision matrix is computed

**Table 3: Weighted Normalized Decision matrix**

| Materials | MOE | YS | UTS | PR | D |
|---|---|---|---|---|---|
| **Steels** | 0.104312134 | 0.188631257 | 0.069261672 | 0.094771103 | 0.087534 |
| **Aluminum and its alloys** | 0.034770711 | 0.094315628 | 0.004617445 | 0.0977327 | 0.029543 |
| **Titanium and its alloys** | 0.064192083 | 0.075452503 | 0.046174448 | 0.100694297 | 0.048691 |
| **Ceramics** | 0.053493402 | 0 | 0.115436119 | 0.059231939 | 0.054709 |
| **Carbon fibers** | 0.160480207 | 0 | 0.09696634 | 0.065155133 | 0.019695 |
| **Kevlar fibers** | 0.037445382 | 0 | 0.129288454 | 0.10661749 | 0.015318 |

MOE=Modulus of Elasticity (E) (GPa), YS=Yield stress (Y) (MPa), UTS=Ultimate Tensile Strength (UTS) (MPa), PR=Poisson's ratio (ν), D=Density (ρ)(kg/m3)

From Table 3, the ideal solution & negative ideal solution are computed by using eqns

$$\boldsymbol{A^+ = \{v_1^+, \dots\dots\dots, v_m^+\} = \{(max v_{ij} \backslash j \epsilon I'), (min v_{ij} \backslash j \epsilon I'')\}}$$
$$\boldsymbol{A^- = \{v_1^-, \dots\dots\dots, v_m^-\} = \{(min v_{ij} \backslash j \epsilon I'), (max v_{ij} \backslash j \epsilon I'')\}}$$

**Table 4: The Ideal solution & Negative ideal solution**

| | | | | | |
|---|---|---|---|---|---|
| $\boldsymbol{A^+}$ | 0.1604802 | 0.1886312 | 0.1292884 | 0.1066174 | 0.087534 |
| $\boldsymbol{A^-}$ | 0.0347707 | 0 | 0.0046174 | 0.0592319 | 0.015318 |

By using Table 4, ideal solution **($D_i^+$)**, and the negative ideal solution **($D_i^-$)** is computed using

$$\boldsymbol{D_i^+ = \sqrt{\sum_{j=1}^{m} \left(v_{ij} - v_j^+\right)^2}, D_i^- = \sqrt{\sum_{j=1}^{m} \left(v_{ij} - v_j^-\right)^2}}$$

**Table 5: Ideal Solution matrix**

| Materials | MOE | YS | UTS | PR | D |
|---|---|---|---|---|---|
| **Steels** | 0.003154852 | 0 | 0.003603215 | 0.000140337 | 0 |
| **Aluminum and its alloys** | 0.015802877 | 0.008895438 | 0.01554286 | 7.89395E-05 | 0.003363 |
| **Titanium and its alloys** | 0.009271403 | 0.01280943 | 0.006907938 | 3.50842E-05 | 0.001509 |
| **Ceramics** | 0.011446176 | 0.035581751 | 0.000191887 | 0.00224539 | 0.001077 |
| **Carbon fibers** | 0 | 0.035581751 | 0.001044719 | 0.001719127 | 0.004602 |
| **Kevlar fibers** | 0.015137568 | 0.035581751 | 0 | 0 | 0.005215 |

MOE=Modulus of Elasticity (E) (GPa), YS=Yield stress (Y) (MPa), UTS=Ultimate Tensile Strength (UTS) (MPa), PR=Poisson's ratio (ν), D=Density (ρ)(kg/m3)

**Table 6: Negative Ideal Solution matrix**

| Materials | MOE | YS | UTS | PR | D |
|---|---|---|---|---|---|
| **Steels** | 0.004836009 | 0.035581751 | 0.004178876 | 0.001263032 | 0.005215 |
| **Aluminum and its alloys** | 0 | 0.008895438 | 0 | 0.001482309 | 0.000202 |
| **Titanium and its alloys** | 0.000865617 | 0.00569308 | 0.001726984 | 0.001719127 | 0.001114 |
| **Ceramics** | 0.000350539 | 0 | 0.012280779 | 0 | 0.001552 |
| **Carbon fibers** | 0.015802877 | 0 | 0.008528319 | 3.50842E-05 | 1.92E-05 |
| **Kevlar fibers** | 7.15386E-06 | 0 | 0.01554286 | 0.00224539 | 0 |

MOE=Modulus of Elasticity (E) (GPa), YS=Yield stress (Y) (MPa), UTS=Ultimate Tensile Strength (UTS) (MPa), PR=Poisson's ratio (ν), D=Density (ρ)(kg/m3)

Compute grey correction degree based on the normalized matrix Y, the grey correction degree of the $i_{th}$ enterprise to the $j_{th}$ indicator of PIS and NIS are:

$$s_{ij}^{+} = \frac{\min_i \min_j |v_{ij}-v_j^{+}| + \rho \max_i \max_j |v_{ij}-v_j^{+}|}{|v_{ij}-v_j^{+}| + \rho \max_i \max_j |v_{ij}-v_j^{+}|},\ s_{ij}^{-} = \frac{\min_i \min_j |v_{ij}-v_j^{-}| + \rho \max_i \max_j |v_{ij}-v_j^{-}|}{|v_{ij}-v_j^{-}| + \rho \max_i \max_j |v_{ij}-v_j^{-}|}$$

$$W^{+} = \begin{bmatrix} s_{11}^{+} & s_{12}^{+} & s_{13}^{+} & \cdots & s_{1n}^{+} \\ s_{21}^{+} & s_{22}^{+} & s_{23}^{+} & \cdots & s_{2n}^{+} \\ s_{31}^{+} & s_{32}^{+} & s_{33}^{+} & \cdots & s_{3n}^{+} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ s_{m1}^{+} & s_{m2}^{+} & s_{m3}^{+} & \cdots & s_{mn}^{+} \end{bmatrix},\ W^{-} = \begin{bmatrix} s_{11}^{-} & s_{12}^{-} & s_{13}^{-} & \cdots & s_{1n}^{-} \\ s_{21}^{-} & s_{22}^{-} & s_{23}^{-} & \cdots & s_{2n}^{-} \\ s_{31}^{-} & s_{32}^{-} & s_{33}^{-} & \cdots & s_{3n}^{-} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ s_{m1}^{-} & s_{m2}^{-} & s_{m3}^{-} & \cdots & s_{mn}^{-} \end{bmatrix}$$

**Table 7: Grey correction degree PIS matrix**

| Materials | MOE | YS | UTS | PR | D |
|---|---|---|---|---|---|
| **Steels** | 0.056168 | 0 | 0.060027 | 0.011846 | 0 |
| **Aluminum and its alloys** | 0.125709 | 0.094316 | 0.124671 | 0.008885 | 0.057991 |
| **Titanium and its alloys** | 0.096288 | 0.113179 | 0.083114 | 0.005923 | 0.038843 |
| **Ceramics** | 0.106987 | 0.188631 | 0.013852 | 0.047386 | 0.032825 |
| **Carbon fibers** | 0 | 0.188631 | 0.032322 | 0.041462 | 0.067839 |
| **Kevlar fibers** | 0.123035 | 0.188631 | 0 | 0 | 0.072215 |

MOE=Modulus of Elasticity (E) (GPa), YS=Yield stress (Y) (MPa), UTS=Ultimate Tensile Strength (UTS) (MPa), PR=Poisson's ratio (ν), D=Density (ρ)(kg/m3)

**Table 8: Grey correction degree NIS matrix**

| Materials | MOE | YS | UTS | PR | D |
|---|---|---|---|---|---|
| **Steels** | 0.069541 | 0.188631 | 0.064644 | 0.035539 | 0.072215 |
| **Aluminum and its alloys** | 0 | 0.094316 | 0 | 0.038501 | 0.014224 |
| **Titanium and its alloys** | 0.029421 | 0.075453 | 0.041557 | 0.041462 | 0.033372 |
| **Ceramics** | 0.018723 | 0 | 0.110819 | 0 | 0.03939 |
| **Carbon fibers** | 0.125709 | 0 | 0.092349 | 0.005923 | 0.004377 |
| **Kevlar fibers** | 0.002675 | 0 | 0.124671 | 0.047386 | 0 |

MOE=Modulus of Elasticity (E) (GPa), YS=Yield stress (Y) (MPa), UTS=Ultimate Tensile Strength (UTS) (MPa), PR=Poisson's ratio (ν), D=Density (ρ)(kg/m3)

$$\xi_i = T_i^{+} \big/ (T_i^{+} + T_i^{-}) \quad i = 1,2, \ldots n$$

Where $\xi_i$ is called best quality score in this study representing the relative approaching degree of the $i_{th}$ enterprise to PIS and NIS

**Table 8: Best Quality Score ($\xi_i$)**

| **Materials** | $\xi_i$ |
|---|---|
| **Steels** | 0.515082042 |
| **Aluminum and its alloys** | 0.414651868 |
| **Titanium and its alloys** | 0.400188733 |
| **Ceramics** | 0.462991119 |
| **Carbon fibers** | 0.472134171 |
| **Kevlar fibers** | 0.442377011 |

From the Best Quality score values the optimal value is finally determined such that the larger value indicates better quality.